DEVELOPMENT OF A DUAL ANALYTICAL/COMPUTATIONAL FRAMEWORK FOR THE DESIGN AND MODELLING OF HYBRID FILAMENT-WOUND AND AFP CARBON FIBER ROLLERS

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Abstract

Fiber reinforced composites are increasingly being utilized in industrial machinery through the replacement of traditional steel rollers with Carbon Fiber Reinforced Polymer (CFRP) alternatives. The design and analysis of these new hybrid components require methodologies capable of accurately predicting their structural response while remaining computationally efficient. In the current work, a dual analytical/computational framework for the modelling of hybrid filament-wound and Automated Fiber Placement (AFP) CFRP rollers is presented. An existing multi-layer coaxial cylinder elasticity solution for laminates of orthotropic layers is expanded to include metallic inserts and residual thermal stresses. A lengthwise solution is achieved through 1D Euler-Bernoulli Finite Element Analysis, allowing reconstruction of the local 3D stress and strain fields, eliminating the need for detailed 3D Finite Element Models. Special consideration is also given to the representation of filament-wound woven ±ϑ layers through equivalent orthotropic unidirectional plies. The proposed methodology is evaluated experimentally through impact hammer modal analysis and 3-point bending tests. Experimental results indicate satisfactory agreement regarding stiffness, modal behavior and failure prediction, with most predictions remaining within a 20% deviation from measured values. The developed framework is shown to provide an effective first-level design tool for hybrid CFRP rollers.

Keywords: *filament winding; carbon fiber rollers; FEA; Euler-Bernoulli beam; residual thermal stresses*

1 Introduction

Fiber reinforced composites have been in commercial use for more than half a century. During this period, their use has remained largely confined to high-performance applications such as aerospace and defense. Recently, other industries such as automotive, wind energy and extreme sports have steadily been replacing metallic with composite components, taking advantage of their superior specific properties and cutting back on operational costs. Industrial machinery represents one of the latest sectors transitioning toward composite structures, by swapping traditional steel rollers with carbon fiber ones. Analyzing the structural behavior of these new hybrid components is crucial and calls for the development of new and refined methodologies that accurately capture the physical response of rollers.

The aim of this work is to propose such a framework by expanding upon pre-existing solutions. A multi-layer coaxial cylinder model is modified and expanded for hybrid metal-CFRP rollers, accounting for metallic flanges and residual thermal stresses. A lengthwise solution is achieved computationally through 1D Finite Element Analysis, allowing for a reconstruction of 3D stress and strain fields. The performance of the proposed methodology is explored through comparison with experimental results.

2 Methodology

There are quite a few studies on the elastic behavior of orthotropic cylinders focusing on a variety of loading profiles, including, axisymmetric, bending, axial, surface traction and others. The main focus when designing rollers is bending and axial loading, so a suitable model would need to include both effects. S. G. Lekhnitskii [1] has studied the elasticity of orthotropic cylinders and has derived analytical solutions for various cases. This formulation has been the basis for many authors when investigating the bending problem [2]. The same theoretical basis is adopted here, along with expansions by C. Jolicoeur and A. Cardou. The leading equations are expanded to include thermal effects and the presence of the metallic inserts. Since the model is being developed mostly for parts manufactured through filament winding, a discussion on how winding parameters correlate to orthotropic elasticity is also included.

2.1 Coaxial Cylinder Model

Regardless of the manufacturing process, the carbon fiber cylinders are assumed to be made by stacking multiple distinct layers of fibers impregnated with resin. Each of these layers is assumed to be orthotropic in its principal directions (fiber direction, in plane direction perpendicular to fibers, through the thickness direction). The orientation of the fiber direction does not, in general coincide with the cylinder lengthwise direction, which results in the following stiffness matrix expression for a generalized off-axis orthotropic layer, expressed in the cylindrical coordinates. See Eq. (1).

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \\ \gamma_{\theta z} \\ \gamma_{rz} \\ \gamma_{r\theta} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0 \\ C_{12} & C_{22} & C_{23} & C_{24} & 0 & 0 \\ C_{13} & C_{23} & C_{33} & C_{34} & 0 & 0 \\ C_{14} & C_{24} & C_{34} & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & C_{56} \\ 0 & 0 & 0 & 0 & C_{56} & C_{66} \end{bmatrix} * \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{\theta z} \\ \tau_{rz} \\ \tau_{r\theta} \end{Bmatrix} \tag{1}$$

Lekhnitskii [1] makes use of Airy stress functions and presents an elasticity solution for one orthotropic cylinder. This solution was refined and reconstructed by C. Jolicoeur and A. Cardou [3] for a laminated cylinder of orthotropic off-axis layers. This approach covers two sets of loading, pure bending and axisymmetric, which can be either axial tension/compression or torsion.
When a metallic part is also included, the equations need to be modified, and a solution can be obtained using the same principal system of differential equations. Equations 2 through 7 give the stress distribution for a metallic layer.

$$\sigma_r = (k \sin \varphi) * \left(2Ar - \frac{2D}{r^3}\right) + 2H + \frac{K}{r^2} \tag{2}$$

$$\sigma_\theta = (k \sin\varphi) * \left(6Ar + \frac{2D}{r^3}\right) + 2H - \frac{K}{r^2} \tag{3}$$

$$\sigma_z = \frac{\varepsilon}{C_{33}} + (k \sin\varphi) * \left(\frac{r}{C_{33}} - \frac{8C_{13}Ar}{C_{33}}\right) - \frac{4C_{13}H}{C_{33}} \tag{4}$$

$$\tau_{r\theta} = -(k \cos\varphi) * \left(2Ar - \frac{2D}{r^3}\right) \tag{5}$$

$$\tau_{rz} = (k \cos\varphi) * \left(E + \frac{F}{r^2}\right) \tag{6}$$

$$\tau_{\theta z} = (k \cos\varphi) * \left(-E + \frac{F}{r^2}\right) + \frac{\theta}{\beta_{44}} r \tag{7}$$

The A, D, E, F, H, K and β_{44} are constants that are calculated based on the overall lamination, stacking sequence and geometry. It should be noted that the existence of the metallic layer affects the stress field of all other layers. In other words, the existence of the metallic part changes the behavior of the whole laminate, further highlighting the fact that a simple superposition of the metallic insert and the laminate alone are inadequate in describing the stress state. If an adhesive film is also included in the model, a similar set of equations will be derived with another set of constants.

Regardless of the inclusion or not of metallic parts or adhesive films, the stress expressions can be properly integrated to get the global force-deformation matrix of the cylinder. See Eq. (8)

$$\begin{Bmatrix} P \\ C \\ M \end{Bmatrix} = \begin{bmatrix} E_{axial}A & B_{12} & 0 \\ B_{21} & G_{torsion}J & 0 \\ 0 & 0 & E_{bending}I \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \theta \\ k \end{Bmatrix} \tag{8}$$

where,

- E_{axial} is the effective axial modulus
- $E_{bending}$ is the effective bending modulus
- $G_{torsion}$ is the effective shear modulus in torsion
- B_{12}, B_{21} are coupling terms between axial loading and torsion

For a complete breakdown of how stress expressions and the global force – displacement matrix are derived, the reader should refer to the work by C. Jolicoeur and A. Cardou [3]

2.2 Residual thermal stresses

Composites commonly employ resins that require some type of oven curing at elevated temperatures. The curing process solidifies the laminate at gelation temperature, which is usually around 90 °C. The anisotropy of the laminate due to the lamination sequence will produce residual thermal stresses in the finalized component. The magnitude of such thermal stresses is often significant and should not be neglected during design. Thermal stresses can be incorporated and studied using the model described above.

The coefficients of thermal expansion of each layer are given in the cylindrical coordinate system as in Eq. (9)

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \\ \gamma_{r\theta} \\ \gamma_{rz} \\ \gamma_{\theta z} \end{Bmatrix} = \Delta T \begin{Bmatrix} a_r \\ a_\theta \\ a_z \\ a_{r\theta} \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

Given a ΔT between gelation temperature and environmental temperature, each layer would deform according to its thermal expansion coefficients and its orientation. The formulation into a solid laminate imposes constraints and forces each ply into either a tensile or compressive state of equilibrium, where a single uniform set of thermal strains, ε, θ and k develop. These thermal strains lead to thermal forces, using the inverse of the constitutive matrix in Eq. (8) and ultimately the development of a residual thermal stress field that should be superimposed to the stress field from pure mechanical loading.

2.3 Filament Winding

In filament winding, parts are manufactured by winding impregnated fibers around a rotating mandrel at a constant angle, that is adjusted per layer. Due to the manufacturing procedure, the layers are not strictly unidirectional but appear in a woven pair of +/- θ degrees. The model presented here assumes unidirectional orthotropic layers. Therefore, adjustments should be made to include the effects of the weave, similar to how plate theories adjust for woven fabrics. First, the layers are decoupled and the woven layer is replaced by a pair of +θ and -θ unidirectional layers, the total thickness of which equals that of the original woven ply. However, inner layers have smaller cross-sectional area and smaller area moment of inertia than outer layers and so, the layer (+ or – θ) that is assumed to be inner would need to be thicker. Essentially, thickness should be chosen so that the following Eq. (10), Eq. (11) and Eq. (12)

$$t_{original} = t_\theta + t_{-\theta} \tag{10}$$

$$A_\theta = A_{-\theta} \tag{11}$$

$$I_\theta = I_{-\theta} \tag{12}$$

Since only 2 free variables are present, not all 3 equations can be satisfied simultaneously. When the loading is expected to be mainly axial, Eq. (12) can be ignored. In any other case, a workaround should be employed. When the laminate contains more than one woven +/- θ layer, the thicknesses of all replacing layers can be adjusted cumulatively. Instead of this, it’s also acceptable that the single woven layer is replaced by 3 unidirectional layers, introducing another free variable, which allows for satisfaction of all 3 criteria, although this approach adds another “phantom” interface and computational complexity.

Regardless of which approach is adopted, the exact effects of the weave on the lamina properties are ignored, as is further discussed in Section 4. In the case of cylinders produced by Automated Fiber Placement, layers are already unidirectional, so the need for modification vanishes.

2.4 Roller Model

The model presented so far assumes a constant load acting on a cylinder of constant properties lengthwise. In typical rollers however, metallic inserts extend for only a small portion of the total length and loads vary in the z direction. To overcome this, a computational solution using Finite Elements is developed.

For the sake of simplicity, it is assumed that the carbon fiber roller has 3 distinct regions, the metallic ends that connect to supports, the metallic insert region and the carbon fiber cylinder, as seen in Figure 1. Each of these regions exhibits a different elastic response. The metallic ends are simple isotropic materials and thus their elastic behavior is described adequately by its modulus of elasticity and shear modulus. For the carbon fiber region and the bonded region, the current model estimates a set of axial, bending and shear moduli, as well as the coupling terms.

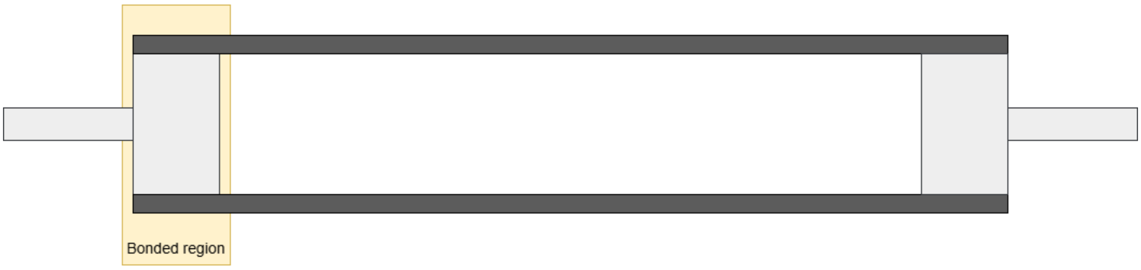

Figure 1. Simplified geometry of a hybrid CFRP-metal roller

Using this assumption, a simple 1D Finite Element Model can be constructed using standard 2-node Euler-Bernoulli elements. This approach is capable of predicting the lengthwise distribution of axial, torsional and bending loads. When a dense enough mesh is selected, these loads can be assumed to remain constant within each element, thus allowing for the use of the stress equations developed previously. Stress intensity factors can also be included if there is a change of diameter between the metallic ends and inserts region or some other discontinuity. For the bonded region, stress intensity will typically be absorbed by the adhesive layer, which requires an analysis of adhesive joints, but relative values of stress concentration factors can be sought in literature. A complete evaluation of stiffness and strength is then achieved

It is noted that in rollers, the bending moment is a result of a transverse force V and not pure bending. Therefore, there will be an additional shear force that can not be examined using the coaxial cylinder framework established here. However, the effects of such a shear force are usually negligible and can be omitted, as is seen in Section 3

The use of other element types such as Timoshenko or higher order shear elements can be explored, although it would require a more complete model beyond pure bending, that predicts a shear factor for the cross-section. Instead, empirical or measured values can be used.

3 Results

To explore the validity of the presented approach, 2 different types of tests were performed: modal analysis using impact hammer testing and 3-point bending.

3.1 Impact Hammer Testing

Various filament wound tubes and rollers of different lamination and materials underwent modal analysis to measure their natural frequencies. In the case of the filament wound tubes, the frequencies were used to measure the axial, torsional and bending stiffness, seen in Table 1, whereas in the case of rollers, the frequencies were compared to results from FE modal analysis, seen in Table 2.

Table 1. Modal analysis results using impact hammer testing. Specimen 1 incorporates pitch carbon fiber. Specimens 2 and 3 incorporate PAN carbon fibers. All specimens are CFRP in epoxy resin

Lamination	**Specimen 1**	**Specimen 2**	**Specimen 3**
	±8/85/±9/85/±10	±45/±46/±47/85/ /±47/±45/±45	$(\pm 25)_2/85/(\pm 26)_2$
Inner Diameter	50 mm	45 mm	90 mm
Outer Diameter	63 mm	53.5 mm	97 mm
Length	6100 mm	1100 mm	5308 mm
Predicted axial modulus	210 GPa	15 GPa	67 GPa
Measured axial modulus	245 GPa	19 GPa	70 GPa
Predicted torsional modulus	11 GPa	33 GPa	20 GPa
Measured torsional modulus	21 GPa	36 GPa	22 GPa
Predicted bending modulus	289 GPa	14 GPa	55 GPa
Measured bending modulus	250 GPa	18 GPa	69 GPa

For measurements in pure composite cylinders, an effort was made to simulate free-free boundary conditions so as to get a clearer signal and be in line with theoretical predictions. For rollers, simply supported conditions were instead preferred, since they are similar to the boundary conditions during service life.

Table 2. Modal analysis results using impact hammer testing on hybrid CFRP-steel rollers. Both rollers incorporate pitch and PAN carbon fibers, epoxy resin matrix and stainless-steel flanges

Lamination	**Roller 1**	**Roller 2**
	±8/85/±9/85/±10	$(\pm 25)_2/85/(\pm 26)_2$
Inner Diameter	110 mm	90 mm
Outer Diameter	120 mm	114 mm
Length	2940 mm	2940 mm
Distance between supports	3200 mm	3200 mm
Diameter at supports	50 mm	50 mm
Bonded region length	80 mm	80 mm
Measured Frequency	50.5 Hz	42.5 Hz
Estimated Frequency	50 Hz	41 Hz

3.2 3-point bending tests

3-point bending experiments were carried out on two specimens to measure the stress-strain curvatures and the total moment until failure. Both specimens were manufactured using PAN carbon fibers and epoxy resin, cured at 120 °C. For the prediction of the elastic behavior, the procedure discussed in Section 2.3 was applied, replacing the woven layers with pairs of +/- θ. The prediction of failure was done assuming only pure bending load (P = 0 = C) and including thermal stresses for a gelation temperature of 90 °C, as shown in Table 3. The EPFS failure criterion was preferred because it gives an overall balance between shear and normal stresses effects [4]. In order to prevent local damage from the machine crosshead and supports, rings cut from another CFRP cylinder were placed at contact points.

Table 3. 3-point bending test results on filament wound cylinders. Both components incorporate PAN carbon fibers and an epoxy resin matrix

Lamination	**Specimen 1**	**Specimen 2**
	±8 / ±8 / 85 / ±8 / ±8	$(\pm8)_3/85/(\pm8)_3/85/(\pm8)_3/85/(30)_2$
Inner Diameter	12 mm	45 mm
Outer Diameter	16 mm	53 mm
Distance between supports	355 mm	950 mm
Predicted bending modulus	88.8 GPa	98.4 GPa
Measured bending modulus	84.4 GPa	92.2 GPa
Predicted Failure Moment	1800 N	13650 N
Measured Failure Moment	2200 N	12430 N

4 Discussion

Experimental results showed a partial agreement to the predicted values using the proposed methodology. Regarding the stiffness estimation, it was observed that axial, torsional and bending moduli are within a 20% error margin of actual values. This margin remains within relative manufacturing uncertainty. Factors, such as fiber volume fraction, misalignment of fibers, voids and the inherent uncertainty regarding the fiber, resin and lamina properties can strongly affect the final properties and could explain such discrepancies to some extent.

Another significant observation can be made regarding specimen 1 that was tested using modal analysis. This was the only specimen that made use of pitch carbon fibers. Pitch carbon fibers are generally more challenging during manufacturing, and the final component is usually of poorer quality than the PAN equivalent. This could contribute further to the lower bending modulus measured. Another 2 important observations are that the bending and axial modulus seem to be very similar for all tested cases, something that is not supported by the developed coaxial model. It is very likely that this is caused by the simplification proposed in Section 2.3 regarding the replacement of woven layers. The weave introduces kinking, voids and apparent friction. Also, the weave pattern should strongly affect the effective properties obtained, although the small number of experiments does not allow for any deduction on the matter.

Additionally, the model assumes only pure bending when forming the constitutive cylinder matrix. If the effects of shear forces were included, it would not only allow for prediction of a shear factor but would possibly affect the calculation of the bending stiffness as well. Timoshenko beam elements could then be incorporated instead of Euler-Bernoulli.

Although the stiffness predictions exhibited deviations from experimental values, the prediction of the first natural frequency of the rollers only exhibited a 2% error, a result which supports the adoption of the presented work as a first level design tool for carbon fiber rollers, despite omitting the adhesive film from the analysis. It should also be noted that both rollers incorporated a combination of pitch and PAN fibers, showcasing that despite uncertainties when monitoring the composite parts individually, roller behavior is accurately predicted.

Finally, the 3-point bending experiments proved the model can be a useful tool in predicting the failure load. Both failure loads were within 20% of the expected values, a margin that is acceptable given the challenges involved in failure prediction. Furthermore, the effect of the chosen failure criterion should not be neglected. Different failure criteria usually predict very different failure loads and the selection of the most appropriate usually involves evaluation of past experience, loading type, material system, geometry and so on.

5 Conclusions

The main objective of the current work was to present a computationally inexpensive, yet robust model for the design of hybrid carbon fiber rollers. The methodology developed, allows for reconstruction of the complete 3D stress field while only using 1D Euler-Bernoulli beam elements and eliminating the need for complex 3D FEA. The approach presented allows for inclusion of thermal residual stresses, failure prediction and stiffness evaluation for common loading profiles. Comparison with experimental results indicated that the model is consistent within the premise of its assumptions. However, the test cases were limited and should be expanded with a greater number of experimental observations. The effects of weave and weaving patterns in particular, seem to strongly affect laminate behavior and should be further studied. The use of Euler-Bernoulli beam elements under a pure bending formulation was shown to produce adequately accurate results. The adoption of higher order shear theories such as Timoshenko beam theory and a modified expression for a combined shear-bending loading are achievable with minor modifications to the existing work. Finally, excluding the adhesive layer proved to be a justified choice, but investigation of its behavior is essential during early stages of design and future work should be focused around it.

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